Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-11-12

Computations

Problem 1. Decide whether $\mathbf{r}_u \times \mathbf{r}_v$ or its negative $\mathbf{r}_v \times \mathbf{r}_u$ points in the correct direction.

(a) Plane $x = u, y = v, z = 4x - 3y$. Direction: upwards.
(b) Cylinder $x = 3\cos u, y = 3\sin u, z = v, 0 \le u \le 2\pi, 0 \le v \le 1$. Direction: outwards.
(c) Sphere $x = 3\sin u \cos v, y = 3\sin u \sin v, z = 3\cos u, 0 \le u \le \pi, 0 \le v \le 2\pi$. Direction: outwards.

Problem 2. Let S be the sphere $x^2 + y^2 + z^2 = R^2$ where R is some fixed positive real number. The surface area of S is $4\pi R^2$.

Using this information (and without parametrizing), compute the flux of $\mathbf{F} = \langle x, y, z \rangle$ outwards through S. That is, compute $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS$.

Hint: How do you find a normal vector for a surface defined by an equation?